DEVELOPMENT OF A REGRESSION MATHEMATICAL MODEL FOR DETERMINING THE THERMAL PERFORMANCE OF A PARABOLIC TROUGH SOLAR COLLECTOR

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Abstract: The article presents the development of a regression mathematical model designed to assess the thermal performance of a parabolic trough solar collector (PTC). The main factors included in the model are the density of the solar flux and the temperature of the coolant. The least squares method was used to build the model, which made it possible to minimize deviations between the measured and predicted values of thermal performance. The result of the work is a regression model that enables highly accurate prediction of the thermal power of the PTC in various climatic and operating conditions. The proposed model can be used to improve the accuracy of forecasts and optimize the operation of the PTC, enhancing its energy efficiency in the design and operation of solar PTCs.

Key words: parabolic trough solar collector, least squares method, optical characteristics, thermal characteristics, adequacy criterion.

Introduction. The development of solar energy and improvement of technologies ensuring efficient conversion of solar energy into thermal energy are important directions for achieving sustainable energy supply. Parabolic trough collectors are widely used in solar thermal installations because they allow efficient concentration of solar radiation onto a narrow focusing line, increasing its density and, consequently, the thermal capacity of the system. One of the key performance indicators of a PTC is its thermal performance, which characterizes the ability of the installation to convert solar radiation into thermal energy for further use [1, 2].

The thermal performance of PTC depends on many factors, such as the angle of incidence of sunlight, geometrical parameters of the concentrator, reflection coefficient of the mirror surface, temperature and flow rate of the coolant, and atmospheric conditions. However, due to the complex and nonlinear dependence of the performance on a number of variables, the development of an adequate mathematical model is a difficult scientific and practical task [3].

To solve this problem, the regression analysis method was used in this work. The regression model based on the least squares method allows to quantitatively describe the influence of various factors on the thermal performance of the PTC and to predict its value with high accuracy under changing operating conditions. The development of such a model requires data collection under different operating conditions of the PTC, statistical analysis to identify significant factors, and optimization of the model parameters to achieve accurate prediction of the thermal capacity [4-5].

Review. The development of mathematical models for predicting the thermal performance of PTCs is an urgent task in the field of renewable energy. In the scientific literature, this topic has been actively studied from various aspects, including theoretical modeling, experimental studies and application of regression analysis. In particular, [6, 7] present numerical models based on finite difference and finite element methods to predict the temperature distribution and energy losses in the system. However, these approaches require significant computational resources and complex parametric studies.

Regression analysis is an effective method for constructing mathematical models that allow describing the dependencies between input and output parameters of the system with high accuracy. Various regression methods including linear and nonlinear regression and machine learning for predicting the thermal performance of solar concentrators have been reviewed in [8, 9]. These studies emphasize the importance of selecting optimal variables and applying statistical model evaluation techniques.

The literature review shows that regression models are a promising tool for analyzing and predicting the performance of PTCs. This paper proposes the development of a new regression model that takes into account key system parameters and their influence on thermal performance to improve the prediction accuracy and optimize the

Methodology. The following mathematical methods are used in this paper:

- regression modeling;
- least squares method;
- linear regression model of heat production capacity of PTC was built;
- Fisher's criterion was used to evaluate the adequacy of the constructed model.

The experiments were conducted on the PTC installed at the polygon of Fergana Polytechnic Institute in the west-east direction.

Results and discussions. The experimental study consists of recording the results of measurement procedures in the hot water treatment process at the PTC. Along with this, the values of key parameters affecting the process efficiency such as, time, water volume, solar radiation, wind speed are determined based on the analysis of experimental data results. In order to clarify the degree of influencing factors, a two-factor regression model determining the thermal performance of PTC was constructed [10].

This paper presents the results of studies of optical and thermal characteristics of the energy module of a solar parabolic trough unit with geometrical parameters: width D=2000 mm, length L=5000 mm, focal length f=866 mm and angle of coverage α=60° (Figure 1).

Figure 1: *General view of a parabolic trough solar collector*

The reflective surface is formed by a mirror sheet of aluminum (produced in Germany) with integral reflection of solar rays R_m=0,79÷0,80. The PTC is installed on the polygon of Fergana Polytechnic Institute in the west-east direction. The concentrator is fixed on two welded trusses providing its installation at a specific angle relative to the horizon calculated for the latitude of the terrain. In the focal plane of the concentrator a receiver in the form of a tube is mounted on adjustable fasteners. The receiver is a stainless-steel pipe with outer diameter d=42 mm, the length of which is l=5000 mm. The heat carrier is water [11-13].

The experiment involves the process of hot water treatment at 80°C for industrial purposes. Table 1 summarizes the results of the experimental data.

Table 1: *Results of the experimental data.*

n	t_e	E_e	T_e
1	0	800	22
2	10	900	33
3	20	988	41
4	30	913	49
5	40	970	58
6	50	540	58
7	60	600	57
8	70	390	55
9	80	1011	68

10	90	990	72
11	100	1035	80

Based on the analysis of experimental data, we construct a two-factor regression mathematical model that determines the thermal performance of the PTC, which has the following form:

$$T_c(t,E) = A_0 + A_1 t_e + A_2 E_e \quad (1)$$

where T_c – water temperature (calculated), t - time, E - solar radiation flux density, A_0, A_1, A_2 – required parameters reflecting the weight contribution of influencing factors, in our case t_e – time (experimental) and E_e – solar radiation flux density (experimental). According to the method of least squares, the difference between the calculated and experimental data should approach zero $\lim_{n\to\infty} \delta^2 = 0$:

$$F = (T_e - T_c) = \delta \Rightarrow (T_e - T_c)^2 = \delta^2$$

$$(T_e - A_0 - A_1 t_e - A_2 E_e)^2 = \delta^2 = 0, \quad (2)$$

where T_e is the water temperature (experimental). From equation (2) we find partial derivatives of the required parameters A_0, A_1, A_2:

$$\begin{cases} \dfrac{\partial F}{\partial A_0} = 2(T_e - A_0 - A_1 t_e - A_2 E_e)(-1) = 0 \\ \dfrac{\partial F}{\partial A_1} = 2(T_e - A_0 - A_1 t_e - A_2 E_e)(-t_e) = 0 \\ \dfrac{\partial F}{\partial A_2} = 2(T_e - A_0 - A_1 t_e - A_2 E_e)(-E_e) = 0 \end{cases}$$

$$\begin{cases} T_e - A_0 - A_1 t_e - A_2 E_e = 0 \\ T_e t_e - A_0 t_e - A_1 t_e^2 - A_2 E_e t_e = 0 \\ T_e E_e - A_0 E_e - A_1 t_e E_e - A_2 E_e^2 = 0 \end{cases}$$

$$\begin{cases} \sum_{i=1}^{N} T_{e_i} - A_0 - A_1 \sum_{i=1}^{N} t_{e_i} - A_2 \sum_{i=1}^{N} E_{e_i} = 0 \\ \sum_{i=1}^{N} T_{e_i} t_{e_i} - A_0 \sum_{i=1}^{N} t_{e_i} - A_1 \sum_{i=1}^{N} t_{e_i}^2 - A_2 \sum_{i=1}^{N} E_{e_i} t_{e_i} = 0 \\ \sum_{i=1}^{N} T_{e_i} E_{e_i} - A_0 \sum_{i=1}^{N} E_{e_i} - A_1 \sum_{i=1}^{N} E_{e_i} t_{e_i} - A_2 \sum_{i=1}^{N} E_{e_i}^2 = 0 \end{cases}$$

From this system of equations, we find the parameters A_0, A_1, A_2 (coefficients of the linear two-factor model) and expression (1) has the following form:

$$T_c(t,E) = -1{,}26 + 0{,}55 t_e + 0{,}032 E_e. \quad (3)$$

Substituting the corresponding values from Table 1 into expression (3) we find the values of $T_c(t,E)$:

Table 2: $T_c(t,E)$ values.

n	$\boldsymbol{t_e}$	$\boldsymbol{E_e}$	$\boldsymbol{T_e}$	$\boldsymbol{T_c}$
1	0	800	22,00	24,34
2	10	900	33,00	34,68
3	20	988	41,00	42,36
4	30	913	49,00	47,45
5	40	970	58,00	56,78
6	50	540	58,00	56,01
7	60	600	57,00	58,94
8	70	390	55,00	53,37
9	80	1011	68,00	70,09
10	90	990	72,00	73,92
11	100	1035	80,00	81,22

Figure 2 shows the changes in water temperature in the PTC absorber tube of the experiment and model.

To be sure that the developed regression model will be applied in practice, we will evaluate its adequacy. Correlation and regression analysis, in our experiments, was carried out on a limited volume. Therefore, the regression and correlation indicators - parameters of the regression equation, correlation coefficient and coefficient of determination can be distorted by random factors. Adequacy shows how much these indicators are characteristic of the entire general population, whether they are not the result of a confluence of random circumstances. To analyze the adequacy of the regression equation (model) we use Fisher's criterion.

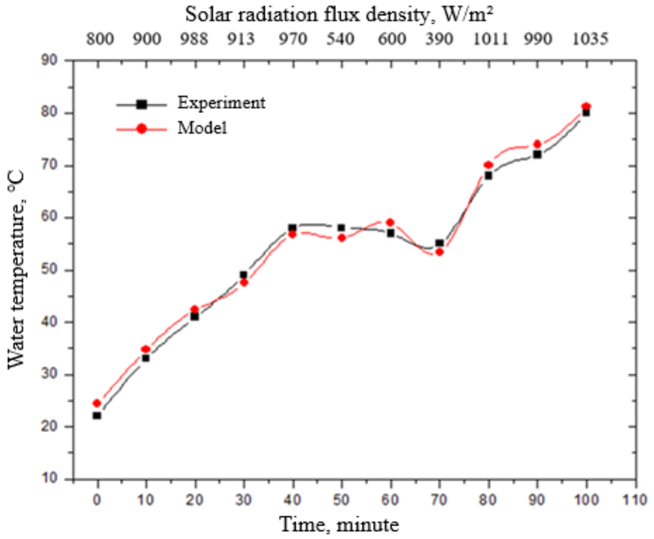

Figure 2: *Variation of water temperature in the PTC absorber tube.*

Checking the significance of the regression equation allows us to establish the strength of fit of the mathematical model, which expresses the relationship between the variables, to the experimental data and indirectly assesses the adequacy of the constructed model. To have a general judgment about the quality of the model, the average approximation error is determined from the relative deviations for each observation. The adequacy of the regression equation (model) is checked by means of the average approximation error, the value of which should not exceed 10% (recommended). The approximation error within less than 5% indicates a good fit of the model to the original data:

$$\bar{\varepsilon} = \frac{1}{n}\sum\frac{|T_e - T_c|}{T_e}\cdot 100\%. \qquad (4)$$

Substituting the experimental data and the corresponding values of the obtained model, we obtained the data shown in Table 3.

Table 3: *Experimental and calculated values of temperature change.*

n	t_e	T_e	T_c	$T_e - T_c$	$(T_e - T_c)/T_e$
1	0	22,00	24,34	2,34	0,106
2	10	33,00	34,68	1,68	0,051
3	20	41,00	42,36	1,36	0,033
4	30	49,00	47,45	1,55	0,032
5	40	58,00	56,78	1,22	0,021
6	50	58,00	56,01	1,99	0,034
7	60	57,00	58,94	1,94	0,034
8	70	55,00	53,37	1,63	0,030
9	80	68,00	70,09	2,09	0,031
10	90	72,00	73,92	1,92	0,027
11	100	80,00	81,22	1,22	0,015
					0,414

Hence,

$$\bar{\varepsilon} = \frac{1}{11}\cdot 0{,}414\cdot 100\% = 3{,}76\%,$$

since, the approximation error is less than 5%, this indicates a good model fit, so the regression equation is adequate.

Assessment of significance of the regression equation as a whole is made on the basis of Fisher's *F*-criterion, which is preceded by analysis of variance. In mathematical statistics, analysis of variance is considered as an independent tool of statistical analysis. To assess significance, we find Fisher's *F*-criterion using the formula

$$F = \frac{\sigma_1^2}{\sigma_2^2}, \qquad (5)$$

where σ_1 and σ_2 are dispersions of samples ($\sigma_1 > \sigma_2$). If $F < F_{crit}$, the hypothesis is accepted. Using the data of Table 2, let us calculate the corresponding dispersions (Table 4).

Table 4: *Dispersions of experimental and calculated values of temperature change.*

t_e	T_e	T_c	$D(T_e)$	$D(T_c)$
0	22,00	24,34	101,82	90,78
10	33,00	34,68	43,72	39,16
20	41,00	42,36	16,66	14,66
30	49,00	47,45	2,41	4,93
40	58,00	56,78	1,67	0,53
50	58,00	56,01	1,67	0,24
60	57,00	58,94	0,96	2,00
70	55,00	53,37	0,12	0,12

80	68,00	70,09	19,86	24,40
90	72,00	73,92	32,73	37,83
100	80,00	81,22	68,07	71,56
sum	**593,00**	**599,16**	**289,69**	**286,21**
average	**53,91**	**54,47**		

Let's evaluate the significance of the obtained regression equation:

$$F = \frac{289{,}69}{286{,}21} = 1{,}01$$

According to the table of Fisher's criterion we find the critical value F_{crit} =4.74, therefore, 4.74>1.01. Since $F_{crit}>F$, the regression equation can be considered adequate.

Taking into account the values of T_e from Table 1 using the formula $Q=cm\Delta t$, where Q – heat capacity, c – specific heat capacity, m – mass, Δt – temperature difference, we determine the heat capacity of PTC. Table 5 shows the averaged values of accumulated energy and solar intensity per unit of time.

Table 5: *Averaged values of stored energy and solar intensity per unit time.*

***n*, number of experiments**	**$\tilde{Q}$, stored energy**	**$\tilde{E}$, solar radiation (averaged per hour)**
1	1238	956
2	1505	1010
3	1254	870
4	920	757
5	1054	814
6	1288	978
7	1505	1012
8	1371	975
9	1037	827
10	1304	892

$$Q = \frac{1}{n}\sum \tilde{Q} = 1247{,}6$$

Generated energy 1247,6/3600*1000=346,5 W, for 10 hours ≈3,5 kW. Figure 3 shows the graph of change in stored energy depending on the change in solar intensity (per unit of time).

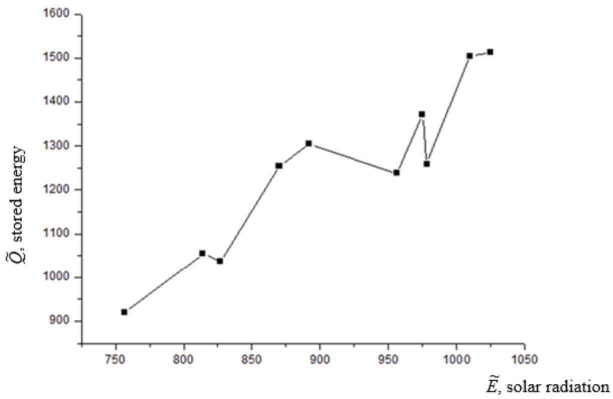

Figure 3: *Stored energy as a function of solar intensity change (per unit time)*

Conclusion. The developed regression model allows to effectively predict the thermal performance of PTC under different operating conditions. The regression equation was tested by Fisher's criterion for adequacy and significance. The adequacy check of the model has shown its sufficient accuracy of applicability in practice. The results of the study can be used to optimize the operation of solar thermal installations, which serves to improve their energy efficiency.

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